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THE APSIDAL MOTION TEST IN 4U 0900–40[1]

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ABSTRACT

Recent *SAS 3* observations of 4U 0900–40 have confirmed the apparent eccentricity of the orbit. Arguments are presented to show that the measured eccentricity is not a spurious effect resulting from systematic reflection of the X-ray beam within the binary system. The longitude of periastron is not observed to have advanced in the 3.5 years since its original determination. The limit on apsidal motion is $\dot{\omega} \lesssim 3\overset{\circ}{.}8 \text{ yr}^{-1}$ which, in turn, restricts the apsidal motion constant to $\log k \lesssim -2.5$. These limits are used to constrain the physical parameters of the companion star HD 77581.

Subject headings: stars: binaries — stars: individual — X-rays: binaries

I. INTRODUCTION

Almost all of our knowledge of stellar interiors is derived from theoretical studies. At present, very few observations can be performed to test any theoretical predictions. Two direct measurements are, however, possible: the measurement of the neutrino flux from the Sun (Davis 1978), and the apsidal motion test for binary stellar systems (Russell 1928). In the former case, one attempts to gain information about the thermal and chemical structure of the solar interior, while in the latter, a one-parameter measure of the mass distribution within a star may be obtained. Reliable measurements of apsidal motion, which results from the tidally and rotationally induced gravitational quadrupole moments of the stars, have been made for about 14 close binary systems (Kopal 1965; see Stothers 1974 for more recent references) wherein both stars are nondegenerate and are usually on the main sequence. The rate at which the longitude of periastron of an eccentric orbit advances can then be related to the "apsidal motion constant" k, which is calculated for various stellar model mass distributions under standard theoretical assumptions (see Schwarzschild 1958). In general, the measured values of k are smaller than the standard calculated values by factors of ~ 2. In all cases, however, the interpretation is complicated by the fact that both stars of the binary system contribute to the apsidal motion.

Binary X-ray pulsars, on the other hand, provide a potentially less ambiguous means of carrying out these measurements. In such systems one of the stars is a neutron star, which acts essentially as a point mass and has a completely negligible apsidal motion constant compared to that of the companion star. There are at present six X-ray binaries for which orbits of the X-ray star have been directly measured (Rappaport *et al.* 1978). Of these, Her X-1 (Tananbaum *et al.* 1972), Cen X-3 (Schreier *et al.* 1972), and SMC X-1 (Primini, Rappaport, and Joss 1977) have highly circular orbits, and 4U 1538–52 is not sufficiently well measured to significantly constrain the eccentricity (Becker *et al.* 1977; Davison, Watson, and Pye 1977). Only 4U 0900–40 = Vela X-1 (Rappaport, Joss, and McClintock 1976; hereafter Paper I) and 4U 0115+63 (Rappaport *et al.* 1978) are appreciably eccentric. The latter is a transient source that was detected in X-rays on only two occasions separated by ~ 7 years. Therefore, 4U 0900–40 is the best candidate system in which to attempt a measurement of the rate of apsidal motion $\dot{\omega}$. In this paper we report the results of such a study. We are able to place a strong upper limit on $\dot{\omega}$ ($\lesssim 3\overset{\circ}{.}8 \text{ yr}^{-1}$) and a correspondingly strong upper limit on $\log k$ ($\lesssim -2.5$). We conclude that this result is marginally consistent with theoretical models for a B0.5 Ib supergiant, which is the spectral type of the optical companion star (HD 77581) in the 4U 0900–40 system (Morgan, Code, and Whitford 1955). In § II of this paper we describe the X-ray observations of 4U 0900–40 and the analysis of the data, in § III we discuss the reality of the orbital eccentricity, and in § IV we discuss the theoretical interpretation and significance of our results.

II. OBSERVATIONS AND ANALYSIS

New pulse timing data from 4U 0900–40 have been obtained during a recent nine-day observation with the *SAS 3* satellite. The y-axis detectors of the X-ray observatory (Buff *et al.* 1977; Lewin *et al.* 1976)

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were pointed nearly continuously at 4U 0900−40 for one complete orbital cycle of the system. For each satellite orbit, the X-ray intensity from the energy channels covering the 3–12 keV range was folded modulo a trial pulse period of ∼283 s. The resultant pulse profiles were each cross-correlated with a master pulse template formed from one orbit of data with especially high signal-to-noise ratio. This technique has been described in several previous papers on X-ray pulsars (e.g., Joss *et al.* 1977; Rappaport *et al.* 1978). After eliminating those times when 4U 0900−40 was in eclipse, when the satellite y-axis drifted off the source, and when data were lost in transmission, we obtained a total of 73 pulse arrival times. Each arrival time is accurate to ∼1.5 s. An approximate relative uncertainty was assigned to each arrival time in order to perform the minimum χ^2 fits described below.

The pulse arrival-time data were fitted to a function of the form

$$t_n = t_0 + P_0 n + \tfrac{1}{2} P_0 \dot{P} n^2 + a_x \sin i \, F(\theta, \omega, \epsilon, \tau) , \quad (1)$$

where t_n is the arrival time of an arbitrary (but fixed) fiducial point on the nth pulse profile, P_0 is the pulse period at a specified epoch, $\dot{P}$ is the rate of change of the intrinsic pulse period, $a_x \sin i$ is the projected semimajor axis of the orbit of the X-ray (neutron) star, and F is the general functional form of an eccentric orbit. The parameters of F are the longitude of periastron ω, the eccentricity e, the time of periastron passage τ, and the mean anomaly θ, where $\theta = 2\pi(t - \tau)/P_{\rm orb}$ and $P_{\rm orb} = 8^{\rm d}.964$ (Ögelman *et al.* 1977) is the orbital period. The Doppler delays resulting from the Earth's motion about the Sun were removed before the fits were performed.

The orbital fit was first carried out with $\dot{P}$ fixed at zero (i.e., under the assumption of no changes in the intrinsic pulse period). The results are shown in Table 1. The fitted parameters are in good agreement with those given in Paper I, as well as those reported by Ögelman *et al.* (1977) and Becker *et al.* (1978). In particular, the value for the eccentricity, $e = 0.094 \pm 0.005$ (1 σ), is statistically significant at the 20 σ level. The longitude of periastron, $158° \pm 5°$ (1 σ), is within the statistical uncertainty of the *SAS 3* value measured in 1975 [$146° \pm 11°$ (1 σ)]. When $\dot{P}$ is allowed to be a free parameter, the eccentricity remains nearly the same but is reduced to the 10 σ statistical confidence level; the fitted value of $\dot{P}/P_0$ is (-0.001 ± 0.002) yr^{-1}, which is both statistically insignificant and an order of magnitude larger than the long-term average value of $\dot{P}/P_0$ observed for this source (Rappaport and Joss 1977*a*). For these reasons, we fix $\dot{P}$ at zero for the remainder of our analysis of each individual observation (but allow for a net change in pulse period between 1975 and 1978, as described below).

The Doppler delays derived from these new observations of 4U 0900−40 are shown in Figure 1. Figure 1*a* displays a fit to a circular orbit (i.e., a fit with e set equal to zero); the residual delays, after subtracting the best-fit circular orbit, show strong evidence for a periodic variation with period $P_{\rm orb}/2$ (as expected if the orbit is actually eccentric). The best-fit eccentric orbit is shown in Figure 1*b*; the residuals from this fit are significantly reduced and appear to be more randomly distributed throughout the orbital cycle.

The 1975 *SAS 3* pulse arrival time data from 4U 0900−40 have been reanalyzed in an attempt to improve the determination of possible apsidal motion. The 35 pulse arrival times given in Paper I were derived from a quick-look analysis of the 1975 June and July observations. The production data from these observations have now yielded a total of 70 pulse arrival times. Our analysis of the production data indicates a pulse phase shift between the 14 arrival times obtained in 1975 June and the 56 arrival times obtained in July, which can be reasonably explained by a small but ambiguous change in intrinsic pulse period between the two observations (see Rappaport and Joss 1977*a* for a discussion and references to such effects in X-ray pulsars). We have therefore chosen to eliminate the June data points from the remainder of our analysis. The orbital parameters obtained from a fit to the improved July arrival times are in satisfactory agreement with the parameters given in Paper I.

TABLE 1

ORBITAL PARAMETERS FOR 4U 0900−40[a]

Parameter	November 1978	Joint Fit 1975/1978 Data
P_0 (s):		
1975	...	282.887 ± 0.003
1978	282.7486 ± 0.0004	282.7492 ± 0.0004
$a_x \sin i$ (lt-sec)	112.3 ± 0.8	113.0 ± 0.8
$f(M)$ ($M_\odot$)	19.0 ± 0.4	19.3 ± 0.4
e	0.094 ± 0.005	0.092 ± 0.005
ω	$158° \pm 5°$	$154° \pm 5°$
τ (JD)	$2,443,823.53 \pm 0.13$	$2,443,823.40 \pm 0.13$
$P_{\rm orb}$ (days)	...[b]	8.9649 ± 0.0002
$\dot{\omega}$ (yr^{-1})	...	$0^\circ.4 \pm 1^\circ.7$

[a] Quoted uncertainties are single parameter 1 σ confidence limits.

[b] $P_{\rm orb}$ fixed at $8^{\rm d}.9643$ (Ögelman *et al.* 1977).

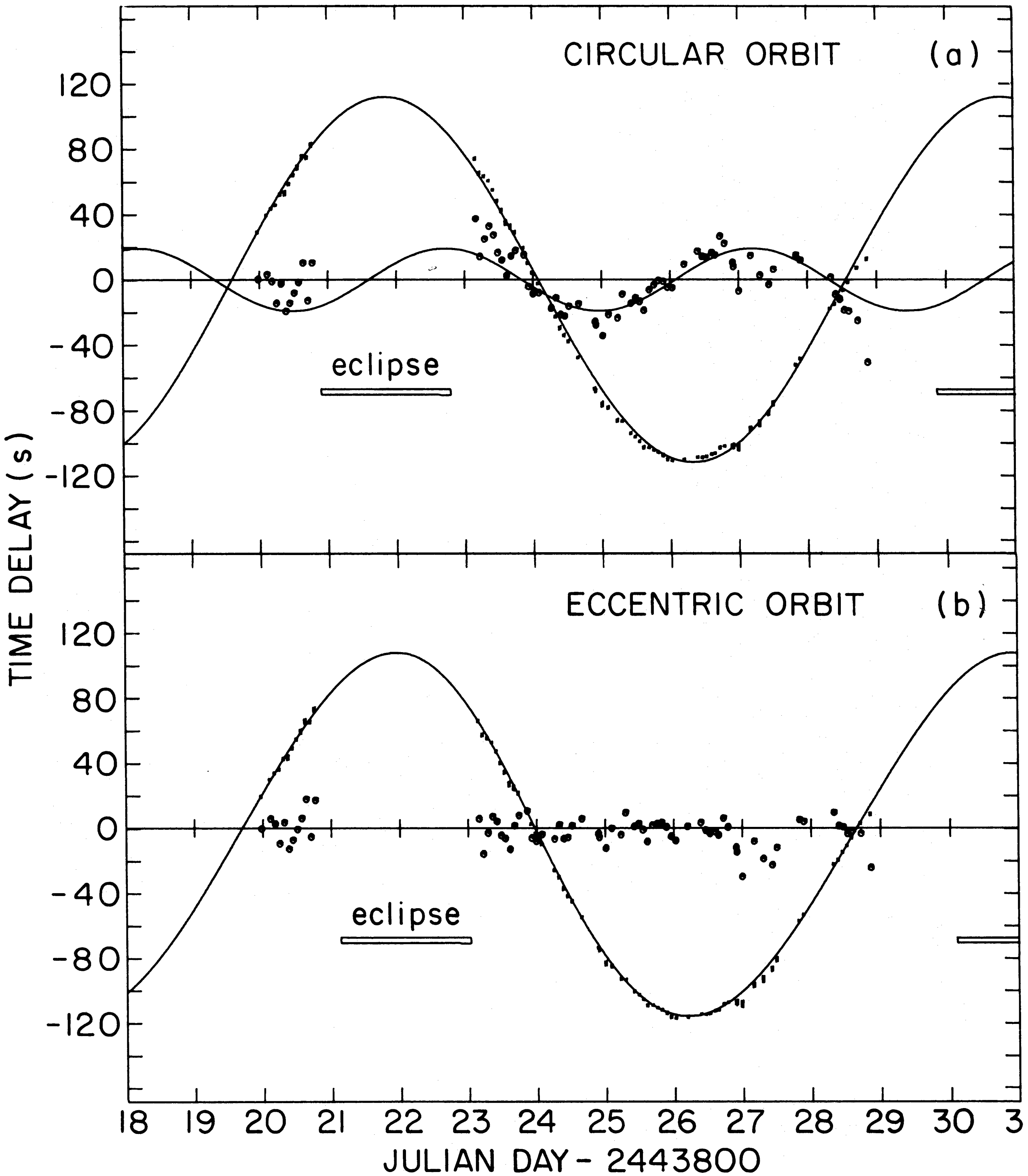

FIG. 1.—Doppler corrections to the 1978 November 4U 0900−40 pulse arrival times as a function of 8ᵈ9649 orbital phase for (*a*) the best-fit circular orbit and (*b*) the best-fit eccentric orbit. The vertical bars are the measured delays; the height of each bar represents $\pm 1\ \sigma$ error limits. The large amplitude solid curves are the best-fit orbits; the circles are the residuals, multiplied by a factor of 5. The small amplitude sinusoidal curve in (*a*), with period $P_{orb}/2$, indicates the systematic trend in the residuals of the circular orbital fit. The indicated eclipse intervals are derived from the best-fit orbital parameters and the representative eclipse duration of 1ᵈ90 that was observed by Forman *et al.* (1973).

The determination of the apsidal motion in 4U 0900−40 was made from an orbital fit to a joint data set comprised of the new 1978 November data and the reanalyzed 1975 July data. This joint data set was fitted to a function of the form:

$$t_{n_j} = t_{0j} + P_{0j}n_j + a_x \sin i\, F(\theta, \omega, \epsilon, \tau, P_{\mathrm{orb}}, \dot{\omega}), \quad (2)$$

where the meaning of the terms is the same as in equation (1) with the addition of $\dot{\omega}$, the rate of apsidal motion (i.e., the rate of change of ω). P_{orb} was now also taken to be a free parameter. The subscript j denotes a particular observation: $j = 1$ for the 1975 data and $j = 2$ for the 1978 data. Thus there are a total of 10 free parameters in the fit. By convention, we have $P_{\mathrm{aps}}{}^{-1} + P_{\mathrm{per}}{}^{-1} = P_{\mathrm{orb}}{}^{-1}$, where $P_{\mathrm{aps}} = 2\pi/\dot{\omega}$ is the period for the longitude of periastron to precess through 360° and P_{per} is the time between successive periastron passages.

The orbital parameters obtained by fitting equation (2) to the joint 1975/1978 data set are given in Table 1. The parameters $a_x \sin i$ and e are in accord with the fits to the individual data sets. The value for P_{orb} [$8^{\mathrm{d}}9649 \pm 0^{\mathrm{d}}0002(1\,\sigma)$] represents a slight improvement over the period determined from optical observations ($8^{\mathrm{d}}966 \pm 0^{\mathrm{d}}001$; Hutchings 1974), and agrees well with the orbital period determinations of Ögelman *et al.* (1977) and Watson and Griffiths (1977). The best-fit value for the rate of apsidal motion is $\dot{\omega} = 0^\circ\!.4 \pm 1^\circ\!.7\ \mathrm{yr}^{-1}$ ($1\,\sigma$), consistent with no apsidal motion. The upper limit (97% confidence) to the apsidal advance during the 3.5 yr interval between *SAS 3* observations is 13°.

III. REALITY OF THE ECCENTRICITY

Before examining the significance of the constraints on apsidal motion, we first address the question of the reality of the eccentricity. Shortly after the publication of Paper I, Milgrom and Avni (1976) raised the possibility that the apparent eccentricity in 4U 0900−40 could result from a circular orbit wherein a portion ($\lesssim 10\%$) of the detected X-ray flux is "reflected" from the primary star. Systematic changes in the amplitude and time delay of the reflected pulse with orbital phase could then mimic an eccentricity.

There are several reasons for now believing that the measured eccentricity is real. The pulse profiles used for the timing analyses are made from X-rays in the 3–12 keV energy range. An inspection of these profiles (Fig. 1 of McClintock *et al.* 1976) reveals that they are highly structured on time scales down to ~ 15 s. The cross-correlation technique used for determining the pulse arrival times makes use of these high-frequency components in the pulse profile. In a recent analysis, Chester (1979) concluded that the use of the higher harmonic content of the 4U 0900−40 pulse profiles considerably reduces the probability that the detected eccentricity is spurious; when the relevant time scale of the pulse structure is shorter than the excess light–travel-time delays of the reflected pulses, one should not observe the systematic shifts in the arrival times needed to produce a significant spurious eccentricity.

To confirm this result, we have performed a numerical simulation of pulse reflection in the 4U 0900−40 binary system. We used an actual pulse profile obtained from 4U 0900−40 to perform the simulation. The reflected pulse was assumed to have the same shape as the direct pulse, but with a phase and amplitude that were allowed to vary. The composite pulse was then analyzed in the same manner as the real data (i.e., cross-correlated with a standard pulse template). We found that because of the high-frequency components in the pulse profile, no significant spurious shifts ($\gtrsim 1$ s) were produced in the arrival times until the amplitude of the reflected pulse was allowed to be $> 30\%$ of the primary pulse amplitude. This is at least a factor of 3 greater than any expected reflection effect in the 4U 0900−40 system (Milgrom and Avni 1976). Finally, we note that the substantial observed eccentricity in the X-ray binary 4U 0115+63 (Rappaport *et al.* 1978) and the probable eccentricity in GX 301−2 (White, Mason, and Sanford 1978) render the measured eccentricity in 4U 0900−40 more plausible. However, it is still possible that reflection effects alter the measured eccentricity in 4U 0900−40 by a small fractional amount.

In principle, fluctuations in the accretion torques on the neutron star in a binary X-ray system could produce drifts in the pulse arrival times (Lamb, Pines, and Shaham 1978) which might also affect the determination of the eccentricity (Boynton 1979). We believe that this is not a serious problem in the case of 4U 0900−40, since approximately the same values of e and ω were measured during four separate observations (Paper I; Ögelman *et al.* 1977; Becker *et al.* 1978; the present work).

IV. THEORETICAL IMPLICATIONS

For a binary system where the orbital and stellar parameters are known, limits on apsidal motion can be used to constrain the values of the stellar apsidal motion constant k (Cowling 1938; Sterne 1939):

$$\dot{\omega} \approx \left(\frac{30\pi g k}{P_{\mathrm{orb}}}\right)\left(\frac{M_x}{M_{\mathrm{opt}}}\right)\left(\frac{R_{\mathrm{opt}}}{D}\right)^5. \quad (3)$$

In this expression M_x, M_{opt}, R_{opt}, and D are the mass of the X-ray star, the mass and radius of the optical companion, and the orbital separation, respectively. The factor g is slightly greater than unity and incorporates the contributions to $\dot{\omega}$ from the non-infinitesimal eccentricity of the orbit and the rotational distortion of the primary; these effects alter the expected value of $\dot{\omega}$ in the 4U 0900−40 system by $< 1\%$ and $< 10\%$, respectively. The contribution to $\dot{\omega}$ from general relativistic effects (Landau and Lifshitz 1962) is also negligible in this system. We take the mass ratio M_x/M_{opt} to be 0.079 ± 0.010 ($2\,\sigma$) and R_{opt}/D to be 0.60 ± 0.05 ($2\,\sigma$) (see the discussion below for references). Our limit on apsidal motion, $\dot{\omega} \lesssim 3^\circ\!.8\ \mathrm{yr}^{-1}$ (97% confidence), yields a limit on $\log k$ of

$$\log k \lesssim -2.5\ (97\%\ \text{confidence}).$$

This limit on $\log k$ is a direct measure of the structure of the companion star HD 77581.

We have calculated values of k for a wide range of stellar models. Companion masses in the range 10–40 $M_\odot$, radii of 20–50 $R_\odot$, and effective temperatures of T_e = 17,000–28,000 K were considered. However, only combinations leading to luminosities that are greater than or equal to the zero-age main-sequence values and less than the Eddington limit were used. The method of generation of the stellar models was as follows. The basic equations of equilibrium stellar structure and Radau's equation for the tidal distortion (Schwarzschild 1958) were integrated from the stellar surface down to a layer just above the hydrogen-burning region. It turns out that, even for those stellar models that have relatively low central condensations, the solution for the tidal distortion is accurately determined by considering only the stellar envelope. Therefore, the complicated (and highly uncertain) prior history of the star does not have to be known. For simplicity, a composition of $(X, Z) = (0.739, 0.021)$ and standard Los Alamos opacities have been adopted. Changes of these parameters would have some effect on our results, but perhaps not more than a factor of 2 in k for reasonable composition changes; however, use of the Carson opacities would provide a substantial decrease in k. (See Odell 1974; Stothers 1974). The results of these calculations are shown in Figure 2 as contours of constant $\log k$ in the $M_{\rm opt}$–$R_{\rm op}$ plane; only the results for T_e = 20,000 K (Fig. 2*a*) and T_e = 22,500 K (Fig. 2*b*) are shown.

We also established a range of allowed parameters for the mass, radius, and effective temperature of HD 77581. The essential observational data are $a_x \sin i$, T_e, the semiamplitude of the optical Doppler velocity curve, $K_{\rm opt}$, and the X-ray eclipse half-angle $\theta_{\rm ecl}$. The values of these parameters that we adopt are: $a_x \sin i = 113.0 \pm 0.8$ lt-sec (present work), $K_{\rm opt} = 21.8 \pm 1.2$ km s^{-1} (van Paradijs *et al.* 1977), $\theta_{\rm ecl} = 31°$–$40°$ (Forman *et al.* 1973; Watson and Griffiths 1977), and $T_e = 22{,}500 \pm 2500$ K (see, e.g., Avni and Bahcall 1975; for additional references, see Nandy and Schmidt 1975; Conti 1978; Underhill *et al.* 1979). The quoted uncertainties for $a_x \sin i$, $K_{\rm opt}$, and T_e are $\sim 1\ \sigma$ confidence limits, while for $\theta_{\rm ecl}$ the quoted range represents extreme limits. The mass and radius of the star HD 77581 can then be calculated from

$$M_{\rm opt} = f(M)(1 + q)^2/\sin^3 i \qquad (4)$$

and

$$R_{\rm opt} = (a_x \sin i)(1 + q)(\cos^2 i + \sin^2 i \sin^2 \theta_{\rm ecl})^{1/2}/\sin i \qquad (5)$$

(see, e.g., Rappaport and Joss 1977*b*), where $f(M)$ is the mass function determined from $a_x \sin i$ and $P_{\rm orb}$, and $q = M_x/M_{\rm opt}$. The deviation from sphericity of the primary star has been neglected in equation (5). The orbital inclination angle i can be estimated from:

$$\sin i \approx [1 - \beta^2(a + b \log q)^2]^{1/2}/\cos \theta_{\rm ecl}\,, \qquad (6)$$

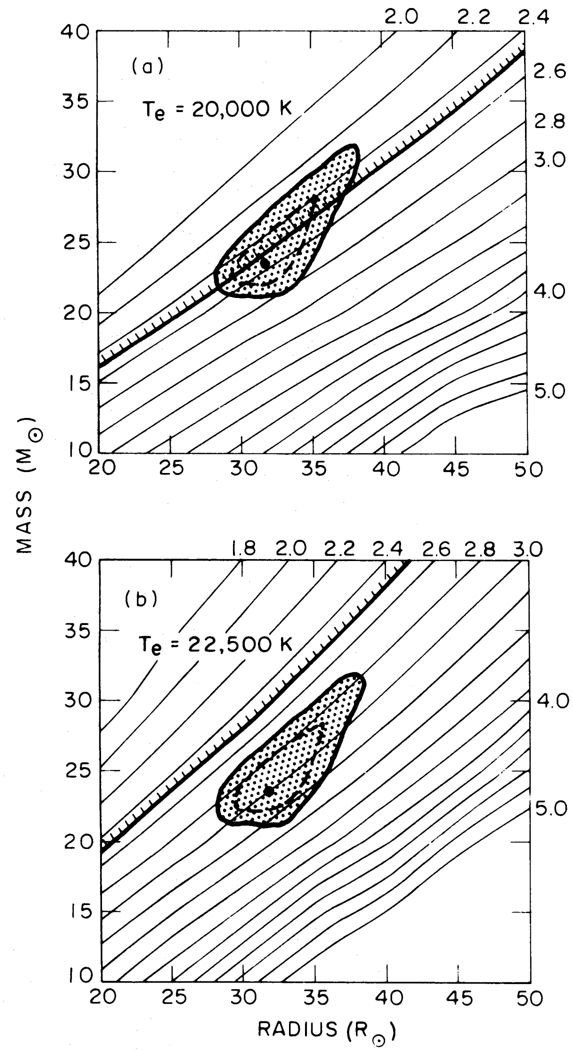

FIG. 2.—Contours of apsidal motion constant, k, in the $M_{\rm opt}$–$R_{\rm opt}$ plane for two assumed values of the effective temperature, T_e. (*a*) T_e = 20,000 K; (*b*) T_e = 22,500 K. The numbers at the top and to the right of each figure denote the value of $-\log k$ for the various contours. The shaded zone represents the error region for the companion star HD 77581 (see text). The heavy curve indicates the measured 97% confidence limit on k and separates the allowed (lower) from the excluded (upper) region.

where β is the fractional radius of the critical potential lobe occupied by the primary and the constants a and b are standard dimensionless parameters used to describe the average radius of the critical potential lobe under the assumption of a specific rotation period, $P_{\rm rot}$, for the primary (e.g., tidal geometry: $P_{\rm rot} = 0$ [Avni 1978] or Roche geometry: $P_{\rm rot} = P_{\rm orb}$ [Paczyński 1971]). (We assume that the orbital and rotational angular velocity vectors are parallel.)

We have evaluated $M_{\rm opt}$ and $R_{\rm opt}$ and their corresponding uncertainties by means of a Monte Carlo error propagation technique. In 2×10^4 trial evaluations, $a_x \sin i$ and $K_{\rm opt}$ were chosen randomly with respect to Gaussian distributions of the appropriate widths. The values of $\theta_{\rm ecl}$ were chosen randomly and uniformly in the range given above. To simulate the theoretical and observational uncertainties in the values of β and $P_{\rm rot}$, we also chose values of β randomly and uniformly in the range 0.9–1.0 and $P_{\rm rot}/P_{\rm orb}$ randomly and uniformly between 0 and 1. The results are shown superposed on the contours of k in Figure 2. The shaded error region for $M_{\rm opt}$ and $R_{\rm opt}$ and the inner region enclosed by the dashed curve contain 98% and 70% of the Monte Carlo events, respectively. The filled circle denotes the most probable parameter values for HD 77581 (i.e., $M_{\rm opt} = 24\ M_\odot$ and $R_{\rm opt} = 32\ R_\odot$), which are in good agreement with those given previously by Paper I and van Paradijs *et al.* (1977). When all of the binary system parameters are evaluated by this technique, the limits on the apsidal motion constant can be found from equation 3. The heavy curve in each of Figures 2*a* and 2*b* separates the allowed from the excluded regions, based on our limit for k (i.e., $\log k \lesssim -2.5$).

The experimental limit on k and our theoretical calculations of k are inconsistent for $T_e \lesssim 18{,}000$ K (not shown in Fig. 2). Thus our results, in principle, constrain the effective temperature of HD 77581 (cf. Nandy and Schmidt 1975, who suggest that the effective temperature of the B0 Ia star ϵ Ori could be as low as $\sim$18,000 K). For $T_e = 20{,}000$ K, the lack of observed aspidal motion is just consistent with theoretical expectations. For $T_e \gtrsim 23{,}000$ K, we should not have observed aspidal motion.

We anticipate that another measurement of $\dot{\omega}$, 3 or more years in the future, should be able to detect the apsidal motion of 4U 0900−40. This will provide a unique measure of the internal mass distribution of an evolved supergiant. Such a measurement will, in turn, yield a direct check on theoretical stellar models and on our understanding of the evolution of close binary stellar systems.

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